Derivation of Poisson Distribution

Start from

$$S_{\upsilon'0} = N_{\upsilon'} N_0 \int_{-\infty}^{\infty} dQ' H_{\upsilon'}\left(Q'\right) e^{-Q'^2} e^{\kappa Q'} e^{-\frac{\kappa^2}{2}}$$

and supplementary sheet on Hermite polynomials, item 2. Use $\lambda = \kappa/2, \quad z = Q' \to Q, \quad \upsilon' \to \upsilon$

$$\Rightarrow S_{\upsilon 0} = N_\upsilon N_0 \int_{-\infty}^{\infty} dQ H_\upsilon\left(Q\right) e^{-Q^2} e^{-\kappa^2/4} \sum_{n=0}^{\infty} \frac{(\kappa/2)^n}{n!} H_n(Q)$$

$$\left[N_\upsilon = \left\{\sqrt{\pi}\ \upsilon!\ 2^\upsilon\right\}^{-\frac{1}{2}}\right]$$

$$= N_\upsilon N_0 e^{-\kappa^2/4} \sum_{n=0}^{\infty} \frac{(\kappa/2)^n}{n!} \quad \frac{\delta_{\upsilon n}}{N_\upsilon N_n}$$

$$= e^{-\kappa^2/4} \quad \frac{(\kappa/2)^\upsilon}{\upsilon!} \quad \sqrt{2^\upsilon \upsilon!}$$

$$\Longrightarrow \boxed{\left|S_{\upsilon 0}\right|^2 = \frac{(\kappa^2/2)^\upsilon}{\upsilon!} \quad e^{-\kappa^2/2}}$$

Poisson Intensity Distribution