Explizite Form der Kugelflächenfunktionen:

$$l = 0: \qquad Y_{00}(\vartheta, \varphi) \equiv \frac{1}{\sqrt{4\pi}} \,,$$

$$l = 1: \qquad Y_{10}(\vartheta, \varphi) = \sqrt{\frac{3}{4\pi}} \cos\vartheta \,,$$

$$Y_{1\,\pm 1}(\vartheta, \varphi) = \mp\sqrt{\frac{3}{8\pi}} \sin\vartheta \, \mathrm{e}^{\pm\, \mathrm{i}\varphi} \,,$$

$$l = 2: \qquad Y_{20}(\vartheta, \varphi) = \sqrt{\frac{5}{16\pi}} \left(3\cos^2\vartheta - 1\right) \,,$$

$$Y_{2\,\pm 1}(\vartheta, \varphi) = \mp\sqrt{\frac{15}{8\pi}} \sin\vartheta \cos\vartheta \, \mathrm{e}^{\pm\, \mathrm{i}\varphi} \,,$$

$$Y_{2\,\pm 2}(\vartheta, \varphi) = \sqrt{\frac{15}{32\pi}} \sin^2\vartheta \, \mathrm{e}^{\pm\, \mathrm{i}2\varphi} \,.$$

Assoziierte Legendre−Polynome

$$P_l^m(z) = (-1)^m \left(1 - z^2\right)^{m/2} \frac{\mathrm{d}^m}{\mathrm{d}z^m} P_l(z) \,; \qquad m \geq 0 \,,$$

$$P_l^{-m}(z) = (-1)^m \frac{(l-m)!}{(l+m)!} P_l^m(z) \,.$$

Legendre−Polynome

$$P_l(z) = \frac{1}{2^l l!} \frac{\mathrm{d}^l}{\mathrm{d}z^l} (z^2 - 1)^l$$